InfoLab

IC611: Database Systems Design

Relational Algebra

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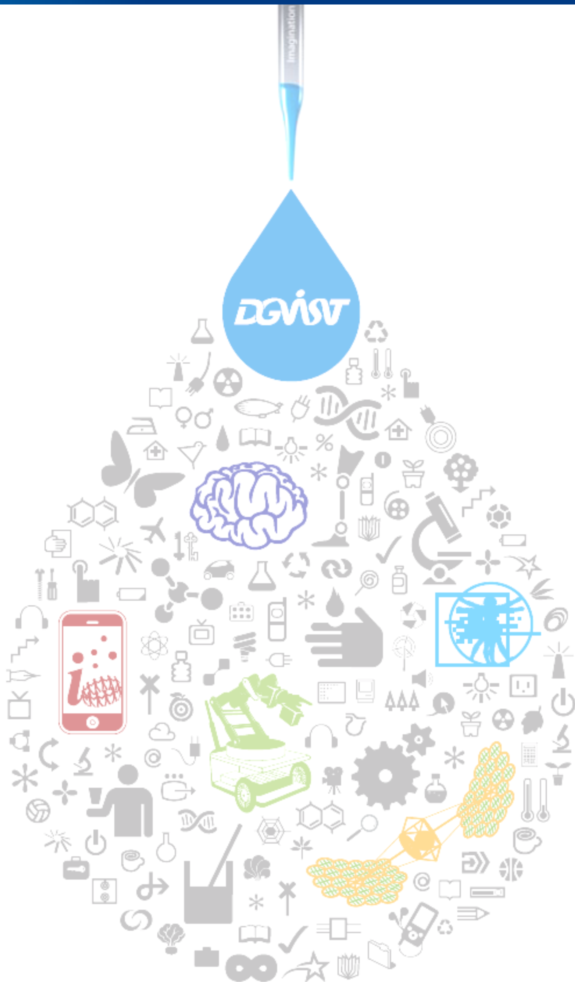

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Relational Query Languages

- **Query languages: Allow manipulation and retrieval of data from a database**

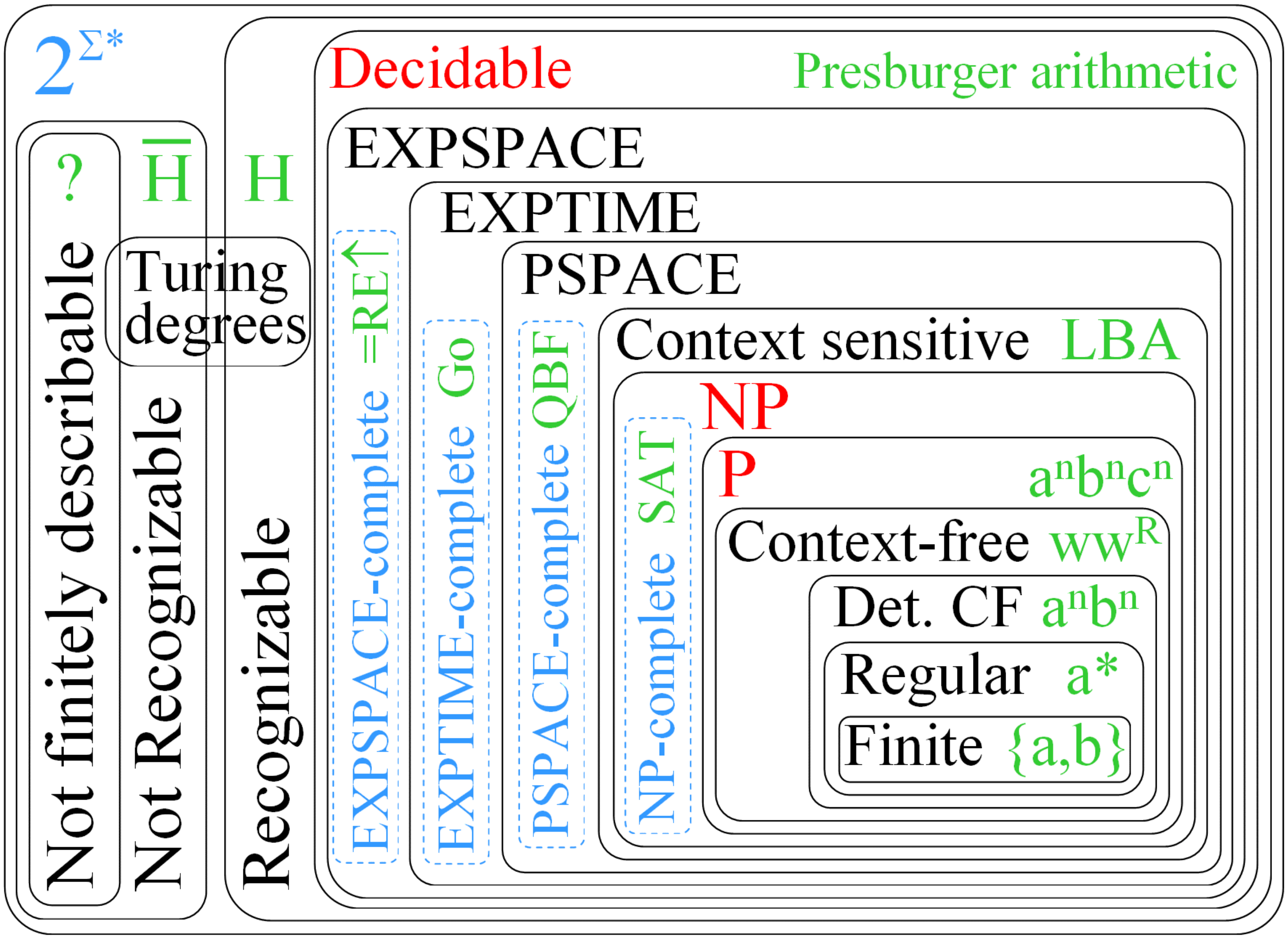
- **Relational model supports simple, powerful QLs:**
  - Strong formal foundation based on logic
  - Allows for much optimization

- **Query Languages != programming languages!**
  - QLs not expected to be “Turing complete”
  - QLs not intended to be used for complex calculations
  - QLs support easy, efficient access to large data sets

Chomsky hierarchy

Grammar	Languages	Automaton	Production rules (constraints)
Type-0	Recursively enumerable	Turing machine	$\alpha \rightarrow \beta$ (no restrictions)
Type-1	Context-sensitive	Linear-bounded non-deterministic Turing machine	$\alpha A \beta \rightarrow \alpha \gamma \beta$
Type-2	Context-free	Non-deterministic pushdown automaton	$A \rightarrow \gamma$
Type-3	Regular	Finite state automaton	$A \rightarrow a$ and $A \rightarrow aB$

The Extended Chomsky Hierarchy



Formal Relational Query Languages

- **Two mathematical Query Languages form the basis for "real" languages (e.g. SQL), and for implementation:**
  - Relational Algebra: More operational, very useful for representing execution plans
  - Relational Calculus: Lets users describe what they want, rather than how to compute it (Non-operational and declarative)

Preliminaries

- **A query is applied to relation instances, and the result of a query is also a relation instance**
  - Schemas of input relations for a query are fixed
    - But, query will run regardless of instance!
  - The schema for the result of a given query is also fixed!
    - Determined by definition of query language constructs

- **Positional vs. named-field notation:**
  - Positional notation easier for formal definitions
  - Named-field notation more readable
  - Both used in SQL

Example Instances

- "Sailors" and "Reserves" relations for our examples
- We'll use positional or named field notation
  - assume that names of fields in query results are `inherited' from names of fields in query input relations

R1

sid	bid	day
22	101	10/10/96
58	103	11/12/96

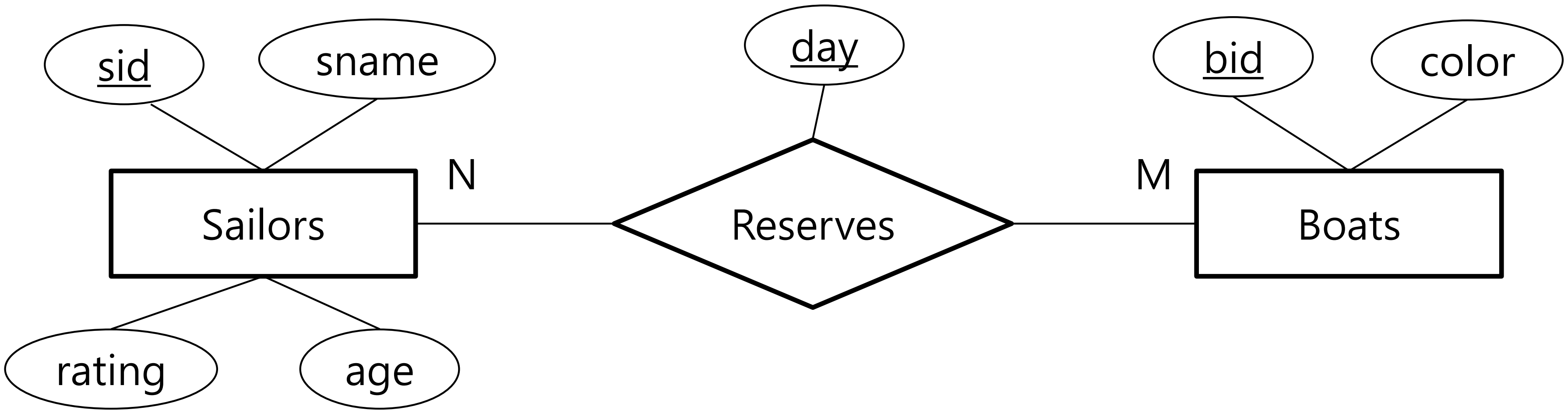
S1

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

S2

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

ER diagram


sid
sname
day
bid
color
N
M
Sailors
Reserves
Boats
rating
age

Relational Algebra

- **Basic operations:**
  - Selection (σ) Selects a subset of rows from relation
  - Projection (π) Extracts wanted columns from relation
  - Cartesian-product ($\times$) Allows us to combine two relations
  - Set-difference ($-$) Tuples in relation 1, but not in relation 2
  - Union ($\cup$) Tuples in relation 1 and in relation 2
- **Additional operations:**
  - Intersection, ***join***, division, renaming: Not essential, but (very!) useful
- **Since each operation returns a relation, operations can be composed! (Algebra is "closed")**

Projection

- **Deletes attributes that are not in projection list**

S2

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

$$\pi_{sname,rating}(S2)$$

sname	rating
yuppy	9
lubber	8
guppy	5
rusty	10

Projection (Cont'd)

S2

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

$\pi_{age}(S2)$

age
35.0
55.5

Projection (Cont'd)

- **Schema** of result contains exactly the fields in the projection list, with the same names that they had in the (only) input relation

- **Projection operator has to eliminate duplicates! (Why??)**
  - Note: real systems typically don't do duplicate elimination unless the user explicitly asks for it (Why not?)

Selection

- Selects rows that satisfy selection condition
- No duplicates in result! (Why?)
- Schema of result identical to schema of (only) input relation
- Result relation can be the input for another relational algebra operation! (Operator composition)

S2

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

$$\sigma_{rating>8}(S2)$$

sid	sname	rating	age
28	yuppy	9	35.0
58	rusty	10	35.0

S2

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

$$\pi_{sname,rating}(\sigma_{rating>8}(S2))$$

sname	rating
yuppy	9
rusty	10

Union, Intersection, Set-Difference

- **All of these operations take two input relations, which must be union-compatible:**
  - Same number of fields
  - Corresponding fields have the same type

- **What is the schema of result (R U S)?**
  - Identical to the schema of R

S1

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

S2

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

$S1 \cup S2$

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0
44	guppy	5	35.0
28	yuppy	9	35.0

S1

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

S2

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

$$S1 - S2$$

sid	sname	rating	age
22	dustin	7	45.0

S1

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

S2

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

$S1 \cap S2$

sid	sname	rating	age
31	lubber	8	55.5
58	rusty	10	35.0

Cartesian-Product (a.k.a. Cross-Product)

- Each row of S1 is paired with each row of R1
- Result schema has one field per field of S1 and R1, with field names `inherited’ if possible

S1

<u>sid</u>	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

R1

<u>sid</u>	<u>bid</u>	<u>day</u>
22	101	10/10/96
58	103	11/12/96

$S1 \times R1$

(sid)	sname	rating	age	(sid)	bid	day
22	dustin	7	45.0	22	101	10/10/
22	dustin	7	45.0	58	103	11/12/
31	lubber	8	55.5	22	101	10/10/
31	lubber	8	55.5	58	103	11/12/
58	rusty	10	35.0	22	101	10/10/
58	rusty	10	35.0	58	103	11/12/

Conflict: Both S1 and R1 have a field called *sid*

Renaming Operator (ρ)

- **ρ (R(F), E) : takes E and return an instance of a (new) relation called R**
  - E is a relational algebra expression E
  - F is a list of terms having the form oldname→newname or position→newname

- **Example: ρ (C(1→ sid1, 5 → sid2), S1 × R1)**

Joins

- **Condition Join**: $R \bowtie_c S = \sigma_c (R \times S)$
- **Result schema same as that of cross-product**
- **Fewer tuples than cross-product**
  - might be able to compute more efficiently
- **Sometimes called a theta-join**

S1

<u>sid</u>	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

R1

<u>sid</u>	<u>bid</u>	<u>day</u>
22	101	10/10/96
58	103	11/12/96

$$S1 \bowtie_{S1.sid < R1.sid} R1$$

(sid)	sname	rating	age	(sid)	bid	day
22	dustin	7	45.0	58	103	11/12/96
31	lubber	8	55.5	58	103	11/12/96

Joins

- **Equi-Join: A special case of condition join where the condition *c* contains only equalities**

- **Result schema similar to cross-product**
  - but, only one copy of fields for which equality is specified

- **Natural Join: Equijoin on all common fields**

  $R \bowtie S$

Relationship among joins

S1

<u>sid</u>	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

R1

<u>sid</u>	<u>bid</u>	<u>day</u>
22	101	10/10/96
58	103	11/12/96

$$S1 \bowtie_{sid} R1$$

sid	sname	rating	age	bid	day
22	dustin	7	45.0	101	10/10/96
58	rusty	10	35.0	103	11/12/96

Reduction and Expansion

- **c.f., Aggregation**

Division

- **Not supported as a primitive operator, but useful for expressing queries like:**
  - Find sailors who have reserved **all** boats

"all" depends on an instance

- **Let A have 2 fields, x and y; B have only field y:**
  - A/B = $\left\{ \langle x \rangle \mid \exists \langle x, y \rangle \in A \ \ \forall \langle y \rangle \in B \right\}$
  - i.e., A/B contains all x tuples (sailors) such that for every y tuple (boat) in B, there is an xy tuple in A
  - or: If the set of y values (boats) associated with an x value (sailor) in A contains all y values in B, the x value is in A/B
- **In general, x and y can be any lists of fields; y is the list of fields in B, and x U y is the list of fields of A**

Examples of Division A/B

sno	pno
s1	p1
s1	p2
s1	p3
s1	p4
s2	p1
s2	p2
s3	p2
s4	p2
s4	p4

pno
p2

B1

pno
p2
p4

B2

pno
p1
p2
p4

B3

sno
s1
s2
s3
s4

A/B1

sno
s1
s4

A/B2

sno
s1

A/B3

Expressing A/B Using Basic Operators

- **Division is not essential op; just a useful shorthand**
  - (Also true of joins, but joins are so common that systems implement joins specially)

- **Idea: For A/B, compute all x values that are not `disqualified' by some y value in B**
  - x value is disqualified if by attaching y value from B, we obtain an xy tuple that is not in A

Disqualified x values: $\pi_x((\pi_x(A) \times B) - A)$

A/B: $\pi_x(A) -$ all disqualified tuples

Find names of sailors who've reserved boat #103

- **Solution 1:** $\pi_{sname}((\sigma_{bid=103} \text{Reserves}) \bowtie Sailors)$

- **Solution 2:** $\rho\ (Temp1,\ \sigma_{bid=103} \text{Reserves})$

  $\rho\ (Temp2,\ Temp1 \bowtie Sailors)$

  $\pi_{sname}(Temp2)$

- **Solution 3:** $\pi_{sname}(\sigma_{bid=103}(\text{Reserves} \bowtie Sailors))$

S1

<u>sid</u>	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

R1

<u>sid</u>	<u>bid</u>	<u>day</u>
22	101	10/10/96
58	103	11/12/96

- **Solution 1:** $\pi_{sname}((\sigma_{bid=103}\,\text{Reserves}) \bowtie Sailors)$
- **Solution 3:** $\pi_{sname}(\sigma_{bid=103}(\text{Re}serves \bowtie Sailors))$

Query Optimization

Find sailors who've reserved a red or a green boat

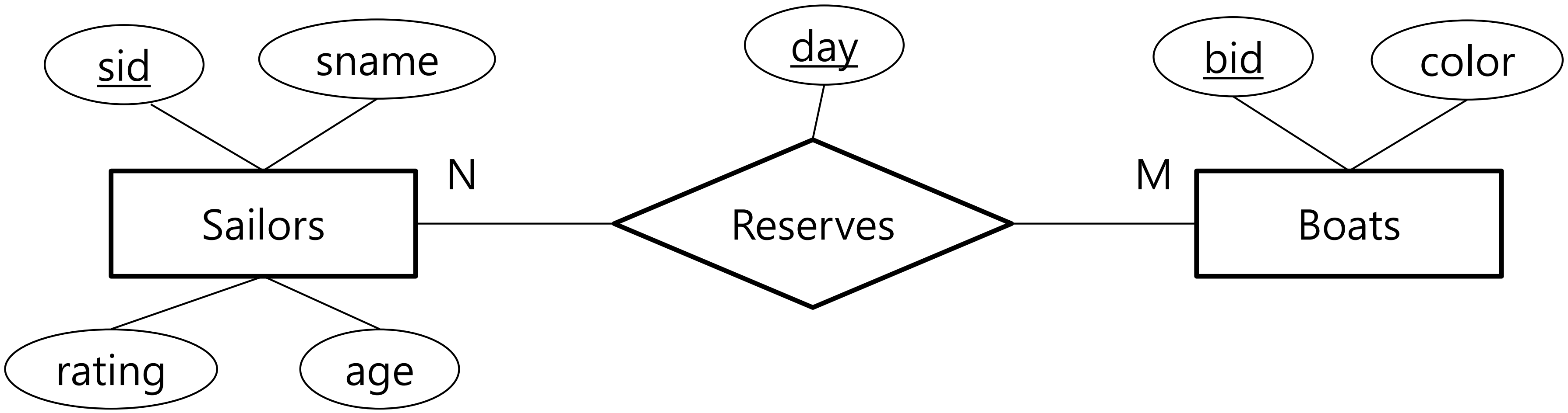
- **Can identify all red or green boats, then find sailors who've reserved one of these boats:**

$$\rho\ (Tempboats, (\sigma_{color='red' \vee color='green'}\ Boats))$$

$$\pi_{sname}(Tempboats \bowtie Reserves \bowtie Sailors)$$

- **Can also define *Tempboats* using union! (How?)**
- **What happens if $\vee$ is replaced by $\wedge$ in this query?**

ER diagram



sid
sname
rating
age
Sailors
N
day
Reserves
M
bid
color
Boats

Find sailors who've reserved a red <u>and</u> a green boat

- **Previous approach won't work!**
- **Must identify sailors who've reserved red boats, sailors who've reserved green boats,**
- **Then find the intersection (note that sid is a key for Sailors)**

$$\rho\ (Tempred, \pi_{sid}((\sigma_{color='red'}\ Boats) \bowtie \mathrm{Re}serves))$$

$$\rho\ (Tempgreen, \pi_{sid}((\sigma_{color='green'}\ Boats) \bowtie \mathrm{Re}serves))$$

$$\pi_{sname}((Tempred \cap Tempgreen) \bowtie Sailors)$$

Find the names of sailors who've reserved <u>all</u> boats

- **Uses division; schemas of the input relations to / must be carefully chosen:**

$$\rho\ (Tempsids, (\pi_{sid,bid} \text{Reserves}) \,/\, (\pi_{bid}\, Boats))$$

$$\pi_{sname} (Tempsids \bowtie Sailors)$$

- **To find sailors who've reserved all 'Interlake' boats:**

$$\ldots\ldots \quad / \ \pi_{bid} (\sigma_{bname='Interlake'}\ Boats)$$

Summary

- The relational model has rigorously defined query languages that are simple and powerful
- Relational algebra is more operational; useful as internal representation for query evaluation plans
- Several ways of expressing a given query; a query optimizer should choose the most efficient version